Effect of Redirectors, Refocusers, and Mode Filters on Light Transmission Through Aberrated and Misaligned Lenses

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The field distortion of a beam propagating through a sequence of identical, misaligned and slightly aberrated lenses is calculated as a perturbation of the Gaussian beam that would propagate in the absence of aberration.

It is found that most of the converted power goes to the first and second modes. They produce deflection and spot-size change of the ideal beam, respectively. The power coupled to modes higher than the second deform the Gaussian profile.

In general, the mode conversion per unit length of guide can be reduced by making the spot size small and by avoiding in-phase coupling at every lens. This last condition is achieved by choosing the period of oscillation of the beam different from an integer number of lens spacings.

Before the beam becomes too distorted, the converted modes must be eliminated. Power in the first and second modes can be reconverted losslessly to the fundamental Gaussian beam by means of servoloops that redirect and refocus the beam. If refocusers are not used, the power in the second mode, as well as the power in the higher-order modes must be absorbed in mode filters such as irises.

For lenses with fourth-order aberration such that at a beam half-width distance from the center the focal length departs δ *percent from ideal, the following typical results are obtained:*

In a guide in which the distance between the beam and guide axes is a constant plus a sinusoid, the converted power is proportional to δ^2*, to the fourth power of the amplitude of the sinusoid and to the square of the number of lenses, but is roughly independent of the curvature of the guide axis.*

On the other hand, in a guide in which the distance between the beam and guide axes is a constant plus a random quantity the converted power is proportional to δ^2*, to the square of the guide curvature, to the mean square of the random deviation, and to the number of lenses.*

For $\delta = 1$ percent, a 1 percent power conversion to the second mode occurs in typical examples, after a few tens of lenses, and the order of magnitude of mode conversion is 0.001 dB/lens. Most of that power is in the second mode and can be recovered with refocusers.

I. INTRODUCTION

Sequences of widely separated glass lenses[1] or periscopic mirrors,[2] as well as sequences of low loss closely spaced gas lenses,[3,4] have been proposed as beam waveguides for long distance optical transmission.

The theory describing the wave and ray propagations through a sequence of misaligned, thin, perfect lenses is known, and those results are applicable even if aberrations are present, provided that the number of lenses is small. Nevertheless, when that number is large, the cumulative effect of lens imperfections must be included.

Before introducing aberrations, though, let us briefly review what is already known about wave and geometric optics in a beam waveguide, assuming throughout the two-dimensional problem instead of the more realistic and complex, but not more enlightening, three-dimensional one.

A paraxial Gaussian beam launched in a periodic sequence of identical thin, aberration-free, but misaligned lenses[5,6,7,8] conserves throughout the Gaussian transverse field distribution. The spot size depends on the initial conditions, the focal length f of the lenses and their spacing L, but is independent of the lens alignment and does not grow with the number of lenses. The geometry of the beam axis, on the other hand, depends also on f, L, and the initial conditions, but more important, it depends on the alignment of the lenses. In general, the beam axis oscillates about the guide axis and the amplitude of the oscillations increases with the number of lenses. For a given set of lenses through which a beam is to be guided, there are then alignment tolerances which must be satisfied in order to prevent the beam from hitting the edges of the lenses. Those tolerances can be drastically alleviated by using redirectors,[9] that is, servoloops that realign the beam axis with the guide axis.

Nevertheless, if the lenses have aberrations, the beam does not conserve the Gaussian profile, but deforms itself[10,11,12,13] as it travels along the guide, the definition of the beam axis then becomes fuzzy, the redirectors become less and less effective, and eventually the grossly distorted beam hits the edges of the lenses.

Gloge[14] has found the effects of random aberrations such as those

which occur both in glass lenses and in the controlled atmosphere between them. Because of the randomness of the aberrations, the beam distortion is independent of the beam trajectory. On the other hand, if all the lenses have the same aberration such as in gas lenses, the beam deformation is strongly dependent on the relation between the beam and the guide axes.

This paper gives an estimate of the beam deformation as a function of systematic aberrations, lens misalignments, and presence or absence of redirectors. It also suggests ways of preventing the beam deterioration together with their price tags.

II. WAVE TRANSMISSION THROUGH SLIGHTLY ABERRATED AND MISALIGNED LENSES

We begin reviewing the wave transmission through ideal lenses and afterward the lenses are slightly perturbed and the mode conversion is calculated.

The wave transmission through a sequence of ideal, thin, equidistant and misaligned lenses as those shown in Fig. 1 is known.[5,6,7,8]

The guide is completely defined by the focal length f of the lenses, their separation L and the radius of curvature R_n of the guide axis at every lens.

The beam axis is characterized by its distance s_n to the guide axis at the nth lens. If the beam is launched through the center of the first lens, it is known[9] that s_n is related to L, f, and R_n by

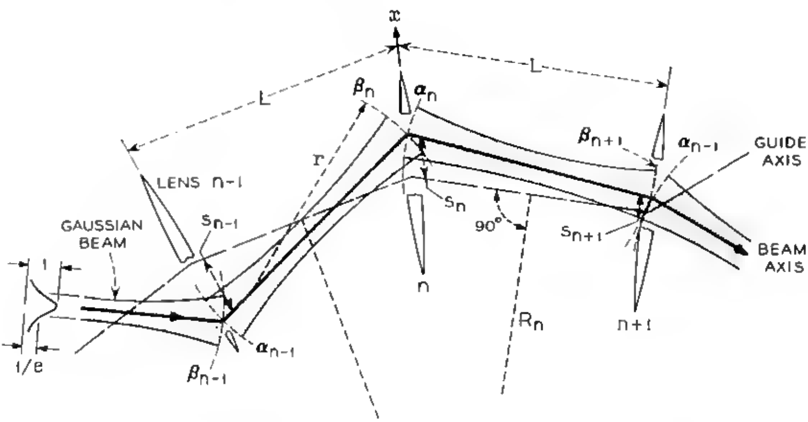

Fig. 1 — Beam transmitted through misaligned ideal lenses.

$$s_n = \frac{L^2}{\sin\theta} \sum_{m=1}^{n-1} \frac{\sin(n-m)\theta}{R_m}, \tag{1}$$

where

$$\cos\theta = 1 - \frac{L}{2f}. \tag{2}$$

The field distribution at every equiphase surface is Gaussian, and its width varies along the beam. Nevertheless, if properly launched, the beam maintains the same half-width w and the same radius of curvature r of the wavefront at every lens. As depicted in Fig. 1, the beam between surfaces α_{n-1} and β_n is the same for all n.

Proper beam launching is achieved if, at the first lens, the radius of curvature of the wavefront is

$$r = 2f, \tag{3}$$

and if the beam half-width w is related to the wavelength λ and the guide parameters in the following manner:

$$w = \sqrt{\frac{\lambda L}{\pi \sin\theta}} = \sqrt{\frac{2\lambda}{\pi} f \tan\frac{\theta}{2}}. \tag{4}$$

Assuming the lenses to be two-dimensional, then the Gaussian beam is also two-dimensional and the electric field measured along the circles α_n or β_n is

$$E = D_0(2\xi) = e^{-\xi^2} \tag{5}$$

and

$$\xi = \frac{x}{w}. \tag{6}$$

Strictly speaking, the normalized length ξ measured along the surfaces α_n or β_n does not coincide with x/w, but if the beam is paraxial, the discrepancy is negligible.

If a higher mode is launched with the same wavefront curvature $1/r$ and the same width w, the field distribution at every surface α_n, β_n is described by the parabolic cylinder function[15]

$$D_p(2\xi) = e^{-\xi^2} He_p(2\xi) \tag{7}$$

in which $He_p(2\xi)$ is the Hermite polynomial of order p. Between the surfaces α_n and β_{n+1}, the phaseshift of the pth mode is $p\theta$ radians smaller than the phaseshift of the fundamental Gaussian mode.

Now let us calculate the phaseshift in passing from the equiphase surface β_n to the equiphase surface α_n, Fig. 2. The length $\overline{AB}$ of a typical ray path between those two wavefronts is

$$\overline{AB} = x\frac{s_n}{f} + \frac{x^2}{r}.$$

Calling

$$\sigma_n = \frac{\bar{s}_n}{w} \tag{8}$$

and using (3), (4), and (8), we obtain

$$\overline{AB} = \frac{\lambda}{\pi}(2\sigma_n\xi + \xi^2)\tan\frac{\theta}{2}.$$

The total phaseshift in passing from surface β_n to α_n is

$$\varphi_n(\sigma_n + \xi) = 2(2\sigma_n\xi + \xi^2)\tan\frac{\theta}{2} + \Phi_n(\sigma_n + \xi). \tag{9}$$

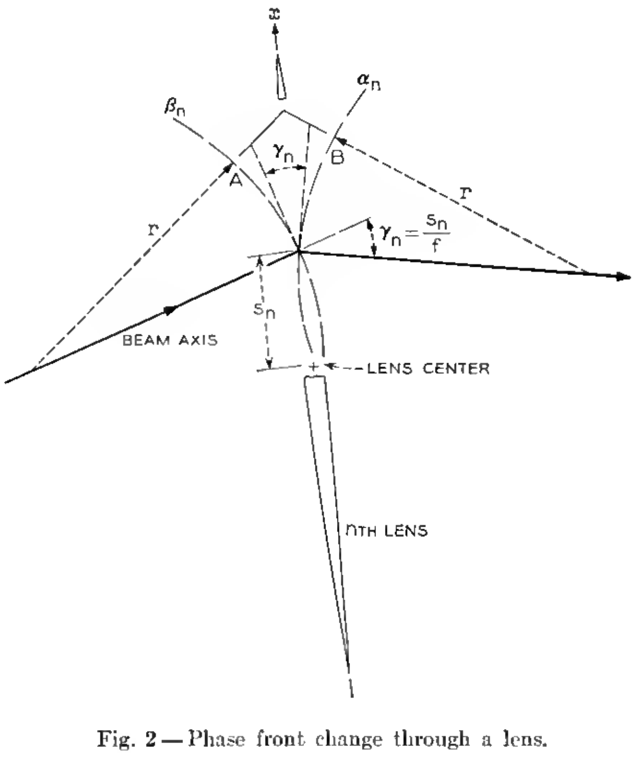

Fig. 2 — Phase front change through a lens.

It is made essentially of two terms. The first is the phase due to the path $\overline{AB}$, the second, $\Phi_n(\sigma_n + \xi)$, is the phase contributed by the lens. Only if the lens is ideal the surface α_n will be an equiphase, and $\varphi_n(\sigma_n + \xi)$ is a constant. The phaseshift through a perfect lens is then

$$\Phi_{n\ \mathrm{ideal}}(\sigma_n + \xi) = \text{const.} - 2(2\sigma_n\xi + \xi^2) \tan \frac{\theta}{2} \tag{10}$$

and since the constant introduces only an uninteresting uniform phaseshift, we will call it zero throughout.

On the other hand, if the lens is not perfect, $\varphi_n(\sigma_n + \xi)$ is not a constant and the field on the surfaces α_n and β_n can no longer be described by a single mode, but rather by a superposition of modes as those given in (7). In general then, the field on the surface α_n is

$$E(\alpha_n) = \sum_{q=0}^{\infty} a_{qn} D_q(2\xi). \tag{11}$$

The amplitude a_{qn} of each mode has been calculated in (57) under the assumptions of small lens distortions and small beam departure from ideal, that is,

$$| \varphi_n(\sigma_n + \xi) | \ll \pi \tag{12}$$

and

$$| a_{qn} | \ll 1 \quad \text{for} \quad q > 0. \tag{13}$$

That amplitude is

$$a_{qn} = \sum_{\nu=1}^{n} c_{oq\nu} e^{iq(\nu-n)\theta} \tag{14}$$

in which $c_{oq\nu}$ is the coupling coefficient between the fundamental and the qth mode at the νth lens, and its value, derived in (58), is

$$c_{oq\nu} = \begin{cases} 1 & \text{if} \quad q = 0, \\ \dfrac{i}{2^q q!} \displaystyle\sum_{\mu=0}^{\infty} \frac{1}{2^{3\mu}\mu!} \frac{\partial^{q+2\mu}\varphi_\nu(\sigma_\nu)}{\partial \sigma_\nu^{\ q+2\mu}} & \text{if} \quad q > 0. \end{cases} \tag{15}$$

Within the approximations involved, the fundamental mode has amplitude one throughout. The amplitude a_{qn} of the qth mode immediately after the nth lens, (14), is made up of n terms. All of them have simple physical meaning. Consider the νth term. The fundamental mode couples $c_{oq\nu}$ into the qth mode of the νth lens and this travels up to the nth lens without further conversion; its phaseshift with respect to the fundamental mode is $q(\nu - n)\theta$.

Since $c_{oo\nu} = 1$, the reconversion into the fundamental mode is not calculated explicitly. Nevertheless, it is automatically taken into account when the power in the higher order modes is ascertained.

The amplitude of the coupled mode a_{qn} can be maintained small by techniques well known from coupled waves theory and which will be used later on:

(*i*) Selecting the phase at the coupling points to provide destructive interference.

(*ii*) Dissipating the power in the unwanted mode.

(*iii*) Providing mode transformers capable of changing unwanted modes into the fundamental one.

III. RECOVERY OF THE FUNDAMENTAL MODE

Physical interpretations of the deformation of a beam traveling through aberrated lenses and ways of preventing that deterioration are considered next.

The field (11) after the nth lens is made essentially by the fundamental mode slightly modified by higher-order modes. Neglecting powers of a_{qn} bigger than one, because of (13), and grouping the first three terms,

$$E(\alpha_n) = (1 - a_{2n})D_0[2\xi(1 - 2a_{2n}) - 2a_{1n}] + \sum_{q=3}^{\infty} a_{qn}D_q(2\xi). \tag{16}$$

The first term is a Gaussian beam different from the ideal one. Its axis is at a distance

$$c_n = \operatorname{Re} a_{1n} \tag{17}$$

from the beam axis of Fig. 1, and its half-width is

$$w_n = 1 + \operatorname{Re} 2a_{2n} . \tag{18}$$

Both dimensions are normalized to the ideal beam half-width w.

Furthermore, the angle between the two axes is

$$\theta_n = \frac{\lambda}{\pi w} \operatorname{Im} a_{1n} \tag{19}$$

and the radius of curvature of the wavefront results

$$r_n = 2f\left[1 - \frac{4}{\tan\dfrac{\theta}{2}} \operatorname{Im} a_{2n}\right]. \tag{20}$$

Expressions (17) through (20) are valid as long as

$$\begin{aligned} | a_{1n} | &\ll 1 \\ | a_{2n} | &\ll 1. \end{aligned} \tag{21}$$

These inequalities can be satisfied for an arbitrarily large number of lenses by periodically realigning the beam and changing its width to the proper size.

The realignment of the beam can be made with redirectors.[9] A feedback servoloop as shown in Fig. 3 senses the position of the beam with photosensors p_1 and p_2 which are centered, for example, on the axis of the pipe in which the lenses are housed.

The difference signal from the sensors is amplified in A and used to displace lens $n - 1$ laterally until the beam axis passes through the center of the photosensors. In general, at every servoloop the beam's axis will pass through the center of the sensor.

The beam size and the curvature of the wavefront can also be adjusted with servoloops which we will call refocusers. The principle of operation is shown in Fig. 4. The beam is aligned with three lenses and three photosensors p_1, p_2, and p_3, are placed at distances from the axis such that an ideal beam would produce equal signals. The difference signal between p_1 and p_2 is amplified in A and controls the focal length of lens $n - 1$, while the difference signal between p_2 and p_3 is amplified in B and used to change the focal length of the nth lens. Once these differences are small, the three signals from the photosensors are practically identical and the beam coincides with the ideal one.

Beam size correction is also possible changing the distance between lenses instead of their focal length.

Obviously, if the lenses are three-dimensional instead of the two-

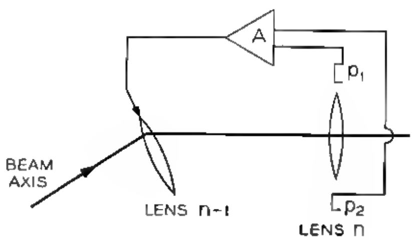

Fig. 3 — Redirector:servoloop for beam realignment.

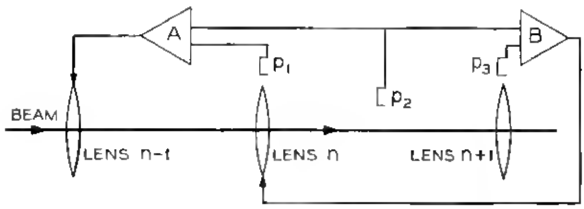

Fig. 4 — Refocuser: servoloop for beam-width adjustment.

dimensional considered above, similar servoloops must be used in two perpendicular directions.

For gas lenses of the tubular type,[3] the beam deflection and focusing may be achieved by dividing the tube in four sectors (Fig. 5) and controlling the temperature T of each of them.[16] If $T_1 = T_2 = T_3 = T_4$, the lens focuses only, but if $T_2 = T_3 = T_4 < T_1$, the lens focuses and deflects the beam downward. If $T_1 = T_3 > T_2 = T_4$, the focusing in the vertical direction is stronger than in the horizontal direction.

For focusing devices such as periscopic mirrors[2] (Fig. 6), the deflection of the beam may be achieved by rotating one or both mirrors around perpendicular axes x and y. As suggested by R. Kompfner, beam refocusing may be obtained by mechanically deforming the mirrors in the two perpendicular directions.

The beam losses in the process of beam refocusing and redirection are due to the interception of the beam by the photosensors, and, in principle at least, they can be made very small indeed. These devices then operate on the idea of reconverting higher unwanted modes into the fundamental.

Unfortunately, it is not simple to make a mode converter capable of taking care of the modes higher than the second contained in the summation in (16). For them and also for the second mode, if refocusers are not used, S. E. Miller suggested another technique which

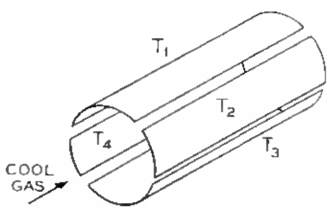

Fig. 5 — Tubular gas lens for beam realignment and refocusing.

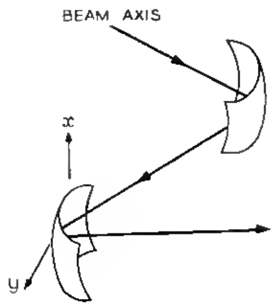

Fig. 6 — Periscopic mirrors.

is essentially lossy, but may be simpler to implement. It consists of using mode filters, perhaps irises aligned with the centers of the redirectors' photosensors.

What are the powers involved in these filtering schemes? At the nth lens, the power in the second mode normalized to that in the fundamental one is

$$P_n^{(1)} = 2 \mid a_{2n} \mid^2. \tag{22}$$

If a_{2n} is real, the radius of curvature of the wavefront (20) coincides with the ideal one $2f$, while the half beam-width departure from ideal results from (18),

$$\mid w_n - 1 \mid = \sqrt{2P_n^{(1)}}. \tag{23}$$

At the same lens, the power carried by the higher-order modes is

$$P_n^{(2)} = \sum_{q=3}^{\infty} \mid a_{qn} \mid^2 q!. \tag{24}$$

There is another filtering scbeme which consists of using at every lens redirectors and filters capable of absorbing the second- and higber-order modes. Tbe power absorbed by the filters in the n first lenses normalized to the power in the fundamental mode is

$$P_n^{(3)} = \sum_{\nu=1}^{n} \sum_{q=2}^{\infty} q! \mid c_{oq\nu}(\sigma_\nu) \mid^2. \tag{25}$$

In tbe following section, several examples are considered and a comparison is made between the powers $P_n^{(1)}$, $P_n^{(2)}$, and $P_n^{(3)}$ to find the most efficient way of avoiding beam deterioration.

IV. EXAMPLES

Let us assume that all the lenses are imperfect but identical, and that the aberration is of fourth order. The phaseshift introduced by the aberration

$$\varphi_\nu(\sigma_\nu) = \delta\sigma_\nu^4 \tag{26}$$

means physically that at a beam half-width w from the center of the lenses, the phaseshift due to aberration is δ radians, while the ideal phaseshift (10) is $-2 \tan \theta/2$. Another physical interpretation is provided by the focal length

$$f(\sigma_\nu) = f\left(1 + \frac{\delta}{\tan \theta/2} \sigma_\nu^2\right) \tag{27}$$

calculated from (10). At a distance w from the center, the focal length is $\delta f/\tan \theta/2$ longer than ideal. For gas lenses,[3] a typical value for δ is 0.01.[17]

Then, the coupling coefficients between the fundamental and the higher-order modes at the νth lens (15) are

$$\begin{aligned} c_{o2\nu} &= i\tfrac{3}{8}\delta(1 + 4\sigma_\nu^2) \\ c_{o3\nu} &= i\frac{\delta}{2}\sigma_\nu \\ c_{o4\nu} &= i\frac{\delta}{16} \\ c_{oq\nu} &= 0 \quad \text{for} \quad q > 4 \end{aligned} \tag{28}$$

and the amplitudes of the different modes after the nth lens (14) turn out to be

$$\begin{aligned} a_{2n} &= i\tfrac{3}{8}\delta \sum_{\nu=1}^{n} (1 + 4\sigma_\nu^2)e^{i2(\nu-n)\theta} \\ a_{3n} &= i\frac{\delta}{2} \sum_{\nu=1}^{n} \sigma_\nu e^{i3(\nu-n)\theta} \\ a_{4n} &= \frac{i\delta}{16} \sum_{\nu=1}^{n} e^{i4(\nu-n)\theta} \\ a_{qn} &= 0 \quad \text{for} \quad q > 4. \end{aligned} \tag{29}$$

To assign values to σ_ν we consider two examples.

4.1 *Beam Guide with Curved Axis*

The lens centers are on a circle of radius R and the beam axis intersects the νth lens at a distance

$$\sigma_\nu = h_0 + h_1 \cos \nu\theta \tag{30}$$

from its center. This means that the beam axis oscillates sinusoidally with amplitude h_1 about a circle of radius $R + h_0 w$.

The constant h_1 depends only in the beam launching conditions, while h_0 is related to the other parameters of the guide[18] by

$$h_0 = \frac{L^2}{4wR \sin^2 \theta/2} = \frac{Lf}{wR}. \tag{31}$$

Substituting (29) in (22) and (24), as well as (26) in (25), assuming θ to be of the order of $\pi/2$ and neglecting terms that do not grow with n, the following powers are derived:

$$\begin{aligned}
P_n^{(1)} &= \tfrac{9}{2}\delta^2 h_1^2\left[h_0 \left|\frac{\sin 3/2n\theta}{\sin 3/2\theta}\right| + \frac{h_1}{4}\left(n + \left|\frac{\sin 2n\theta}{\sin 2\theta}\right|\right)\right]^2 \\
P_n^{(2)} &= \tfrac{3}{2}\delta^2\left[\left(h_0 \frac{\sin 3/2n\theta}{\sin 3/2\theta}\right)^2 + \left(\frac{h_1^2}{4} + \frac{1}{16}\right)\frac{\sin^2 2n\theta}{\sin^2 2\theta}\right] \\
P_n^{(3)} &= \tfrac{3}{8}\delta^2\left[1 + 10\left(h_0^2 + \frac{h_1^2}{2}\right) + 12h_0^4 + 36h_0^2h_1^2\right. \\
&\qquad \left. + \tfrac{3}{2}h_1^4\left(3 + \left|\frac{\sin 2n\theta}{\sin 2\theta}\right|\right) + 12h_0h_1^3\left|\frac{\sin 3/2n\theta}{\sin 3/2\theta}\right|\right]n.
\end{aligned} \tag{32}$$

To minimize these quantities, one must choose the distance L between lenses in such a way that θ is different enough from $\pi/2$ and $2\pi/3$ as to satisfy the inequalities

$$\left|\theta - \frac{\pi}{2}\right| \gg \frac{\pi}{2n}$$

and

$$\left|\theta - \frac{2\pi}{3}\right| \gg \frac{2\pi}{3n}. \tag{33}$$

This choice prevents the systematic in-phase coupling of higher-order modes at every lens. This result can be extended to guides of identical lenses with any aberration. The separation L must be chosen in such a way that the period of oscillation of the beam about the axis does not coincide with an integer number of lens spacings.

If (33) are satisfied, the powers of (32) become

$$P_n^{(1)} = \tfrac{9}{32}\delta^2 h_1^4 n^2 \tag{34}$$

$$P_n^{(2)} = 0 \tag{35}$$

$$P_n^{(3)} = \tfrac{3}{8}\delta^2\left[1 + 10\left(h_0^2 + \frac{h_1^2}{2}\right) + 12h_0^4 + 36h_0^2h_1^2 + \tfrac{9}{2}h_1^4\right]n. \tag{36}$$

Since $P_n^{(2)} = 0$, there is no build-up of power in the third and fourth modes. The beam maintains a Gaussian profile and can be refocused without any power loss.

The power in the second mode grows proportionally to the square of the number of lenses and to the fourth power of the amplitude of the beam axis oscillations, but is independent of h_0 and consequently independent of the radius of curvature R of the guide.

If absorbing mode filters are used, one observes that, while $P_n^{(1)}$ grows proportionally to n^2, (34), $P_n^{(3)}$ grows only proportionally to n. There is cross-over at a number of lenses n_0 for which $P_{n_0}^{(1)} = P_{n_0}^{(3)}$. It is

$$n_0 = \frac{4}{3h_1^4}\left[1 + 10\left(h_0^2 + \frac{h_1^2}{2}\right) + 12h_0^4 + 36h_0^2h_1^2 + \tfrac{9}{2}h_1^4\right]. \tag{37}$$

If $n < n_0$ it is less power consuming to have one filter every n lenses. If $n > n_0$ it is better to use filters at every lens.

For

$$h_0^2 = 1,^*$$

$$h_1^2 = 1, \quad \text{and}$$

$$\delta = 0.01,$$

we calculate from $P_n^{(1)}$ in (34) that the power converted to the second mode is 1 percent after 19 lenses.

Furthermore, one mode filter at the 19th lens, or filters at every lens, would dissipate, respectively,

$$P_{19}^{(1)} = 0.01 \qquad \text{equivalent to } 0.0023 \text{ dB/lens}$$

and

$$P_{19}^{(3)} = 0.049 \qquad \text{equivalent to } 0.011 \text{ dB/lens.}$$

* This value $h_0 = 1$ is derived from (31) using the following typical quantities:

$$L = 0.7 \text{ m}$$
$$R = 1 \text{ km}$$
$$w = 0.5 \text{ mm}$$
$$\theta = \pi/3.$$

It is roughly five times less power consuming to use one filter every 19 lenses. This occurs because the conversions at successive lenses have enough phaseshift to interfere destructively and reduce the converted power level from 0.011 dB/lens to 0.0023 dB/lens.

Given the lenses with aberrations and a length of guide D, is there less mode conversion crowding the lenses or keeping them far apart? To answer this question, (34) is rewritten substituting for n the ratio D/L,

$$P_{D/L}^{(1)} = \left(\frac{9}{32}\frac{\delta^2 h_1^4}{\tan^2 \theta/2} D^2\right)\frac{\tan^2 \theta/2}{L^2}. \tag{38}$$

Because of the normalizations (8), (27), (30) to the ideal beam size w, the parenthesis is a constant and $P_{D/L}^{(1)}$ is minimized by making $\tan^2 \theta/2/L^2$ as small as possible. From (2)

$$\frac{\tan^2 \theta/2}{L^2} = \frac{1}{L^2(4f/L - 1)}.$$

This expression and consequently the power $P_{D/L}^{(1)}$ is independent of the wavelength λ and can be minimized by choosing the separation of the lenses as close to confocal as possible without violating the inequality (33).

Following a similar line of thought one deduces from (32) that the power $P_n^{(2)}$, in modes higher than the second, is reduced by choosing λ as small as possible. This result is illustrated next via a computer experiment[10] that goes beyond the limits of applicability of the perturbation analysis developed in this paper.

Consider a sequence of aberrated, aligned, two-dimensional lenses of width $2a$, spacing L, and focal length

$$f = f_0\left[1 + 0.02\left(\frac{s}{a}\right)^2\right]. \tag{39}$$

A Gaussian beam of half-width w, which is the correct spot size for a sequence of ideal lenses of focal length $f = f_0$, enters parallel to the guide axis at a distance $a/3$. Assuming

$$L = 2f_0 \quad \text{(confocality)}$$

and

$$\frac{w}{a} = \frac{1}{3},$$

the distorted power beam profiles at the 167th and 168th lenses are illustrated in Fig. 7(a).

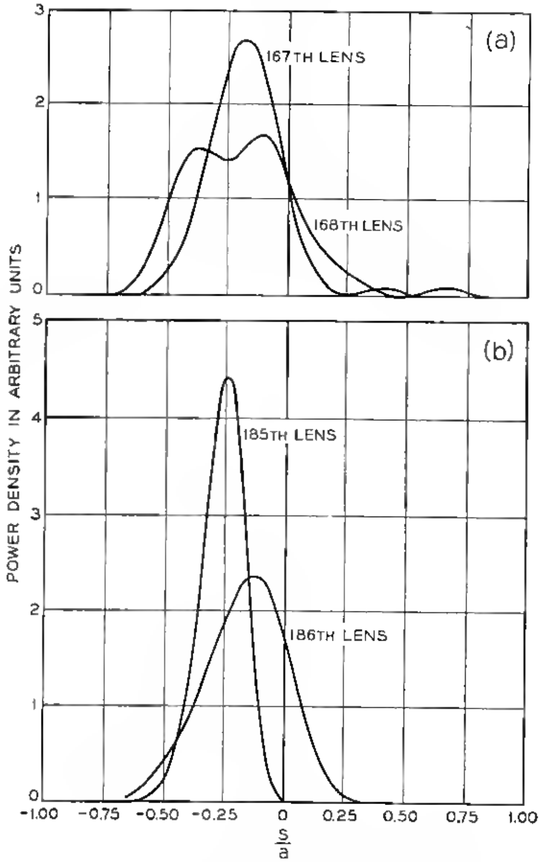

Fig. 7—Power beam profile after many lenses of focal length $f = f_0[1 + 0.02(s/a)^2]$. (a) $L = 2f_0$ (confocal lenses); $w/a = \frac{1}{3}$. (b) $L = 1.8f_0$ (10% off confocal); $w/a = 1/3\sqrt{2}$ (shorter wavelength).

Conversion to distorting high-order modes is substantially reduced by avoiding confocality and by reducing the beam width. For example, assuming

$$L = 1.8f_0$$

and

$$\frac{w}{a} = \frac{1}{\sqrt{2}\,3}$$

the power beam profiles after the same length of guide, Fig. 7(b), are still close to Gaussian.

4.2 *Beam Axis Randomly Dispaced from a Circle*

This beam axis occurs, for example, in the following beam guide. Assume a metallic tube in which the lenses are housed and whose axis is a circle of radius R. With each lens there is a redirector rigidly connected with the tube. To keep the beam away from the wall, the photosensors' centers should coincide with the tube axis, but they don't because of alignment tolerances. At the nth photosensor, their separation is a Gaussian random length d_n, Fig. 8, which we have normalized to the beam half-width w.

The beam axis forced to pass through the center of every photosensor will also depart from the tube axis d_n.

From Fig. 8, one finds that d_n, R, L, and f are related to the normalized distance σ_n between beam axis and lens center by the expression

$$\sigma_n = 2\, d_n - d_{n-1} - d_{n+1} + h_0 \tag{40}$$

in which h_0 is once more the constant defined in (31).

Since d_n is a Gaussian random variable, it follows from (35) with obvious nomenclature

$$\langle\sigma\rangle = h_0 \tag{41}$$

$$\langle\sigma^2\rangle = 6\langle d^2\rangle + h_0^2 \tag{42}$$

$$\langle\sigma^4\rangle = 108\langle d^2\rangle^2 + 36h_0^2\langle d^2\rangle + h_0^4\,. \tag{43}$$

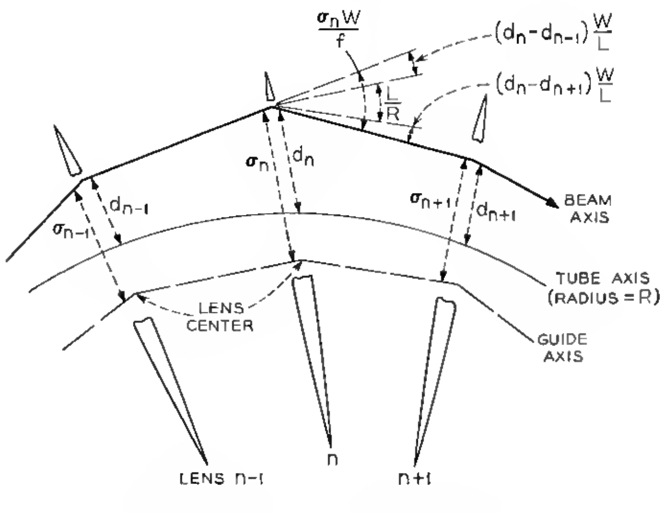

Fig. 8—Beam axis at random distance d_n from a circle of radius R. $\sigma_n = 2d_n - d_{n-1} - d_{n+1} + h_0$; $h_0 = Lf/wR$.

Now we calculate from (22), (24), and (29) the expected power in the second mode and in the higher-order modes at the nth lens

$$\langle P_n^{(1)} \rangle = \tfrac{9}{2}\delta^2(\langle\sigma^4\rangle - \langle\sigma^2\rangle^2)n \tag{44}$$

$$\langle P_n^{(2)} \rangle = \tfrac{3}{2}\delta^2\left[(\langle\sigma^2\rangle - \langle\sigma\rangle^2)n + \frac{1}{16}\left(\frac{\sin 2n\theta}{\sin 2\theta}\right)^2 + \left(h_0 \frac{\sin 3/2n\theta}{\sin 3/2\theta}\right)^2\right]. \tag{45}$$

The expected power to be absorbed by mode filters at every lens is deduced from (25) and (28). It is

$$\langle P_n^{(3)} \rangle = \tfrac{3}{8}\delta^2(1 + 10\langle\sigma^2\rangle + 12\langle\sigma^4\rangle)n. \tag{46}$$

More explicit results are obtained substituting in the last three expressions the averages $\langle\sigma\rangle$, $\langle\sigma^2\rangle$, and $\langle\sigma^4\rangle$ with their equivalents in (41) through (43):

$$\langle P_n^{(1)} \rangle = 108\delta^2\langle d^2\rangle(h_0^2 + 3\langle d^2\rangle)n \tag{47}$$

$$\langle P_n^{(2)} \rangle = 9\delta^2\langle d^2\rangle n + \frac{3}{32}\left(\delta\frac{\sin 2n\theta}{\sin 2\theta}\right)^2 + \frac{3}{2}\left(\delta h_0 \frac{\sin 3/2n\theta}{\sin 3/2\theta}\right)^2 \tag{48}$$

$$\langle P_n^{(3)} \rangle = \tfrac{3}{8}\delta^2[1 + 10h_0^2 + 12h_0^4 + 12\langle d^2\rangle(36h_0^2 + 5) + 1296\langle d^2\rangle^2]n. \tag{49}$$

The power in the second mode grows proportionally to the number of lenses, and if $h_0^2 \gg 3\langle d^2\rangle$, is proportional to the mean square displacement and also proportional to h_0^2.

To prevent build-up of $\langle P_n^{(2)}\rangle$ proportionally to n^2, it is necessary to avoid choosing $\theta = 2\pi/3$ or $\theta = \pi/2$.

In general, $\langle P_n^{(2)}\rangle \ll \langle P_n^{(3)}\rangle$; therefore, it is less power consuming to use beam refocuser and mode filters after several lenses and not at every lens.

For

$$h_0^2 = 1$$

$$\delta = 0.01$$

$$\sqrt{\langle d^2\rangle} = 0.1$$

an expected power conversion to the second mode of 1 percent occurs after 90 lenses. At that lens (47), (48), and (49) become

$\langle P_{90}^{(1)}\rangle = 0.01$	equivalent to 0.00048 dB/lens
$\langle P_{90}^{(2)}\rangle = 0.0008$	equivalent to 0.00004 dB/lens
$\langle P_{90}^{(3)}\rangle = 0.095$	equivalent to 0.0047 dB/lens.

If only mode filters every 90 lenses are used, the loss is $\approx$0.00052 dB/lens. If beam refocuser and mode filters are used every 90 lenses, the loss is 12 times smaller, 0.00004 dB/lens.

Differently from the previous example, the conversion per unit length is reduced by choosing both the separation L between lenses and the wavelength λ as short as possible.

V. CONCLUSIONS

A beam transmitted through few tens of identical misaligned and aberrated lenses becomes distorted due to coupling to unwanted higher-order modes. Unless the beam is restored to ideal shape, the distortion continues until power is lost through the edges of the lenses.

In general, mode conversion per unit length of guide is minimized but not eliminated: (*i*) by choosing the distance between lenses such that the period of oscillation of the beam does not coincide with an integer number of lens spacings; (*ii*) by reducing the spot size, that is, by using short lens spacing and wavelength.

Most of the converted power goes to the first and second modes. They change the beam path and the beam size, respectively. In principle, both can be corrected with negligible loss by means of servomechanisms which redirect and refocus the beam.

Power converted to higher modes than the second distort the wavefront and must be absorbed by mode filters such as irises, for example.

For lenses with fourth-order aberration such that at a beam half-width from the center the focal length is 1 percent shorter than on axis, a 1 percent power conversion to the second mode occurs after few tens of lenses. Mode filters every few tens of lenses restore the original beam with losses of the order of 0.001 dB/lens. If refocusers and filters are used simultaneously, the second-order mode power is recovered and the losses are reduced by one order of magnitude.

Mode filters at every lens are, in general, lossier.

Long distance transmission through aberrated lenses such as our present form gas lenses seems possible, but it hinges heavily on our ability to build efficient and reliable redirectors, refocusers, and mode filters.

APPENDIX

Field in a Sequence of Distorted Lenses

The field on the surface α_n (Fig. 1) is made of a superposition of normal modes

$$E(\alpha_n) = \sum_{q=0}^{\infty} a_{qn} D_q(2\xi). \tag{50}$$

This field is related to the field $E(\beta_n)$ on the surface β_n by the phaseshift $\varphi_n(\sigma_n+\xi)$ given in (9). Thus,

$$E(\alpha_n) = E(\beta_n) \exp\,[i\varphi_n(\sigma_n + \xi)]. \tag{51}$$

Furthermore, the field $E(\beta_n)$ on the surface β_n is related to that on the surface α_{n-1} through the phaseshift of every mode,

$$E(\beta_n) = \sum_{q=0}^{\infty} a_{qn-1} \exp\,(-iq\theta) D_q(2\xi). \tag{52}$$

Substituting (50) and (52) in (51), we obtain

$$\sum_{q=0}^{\infty} a_{qn} D_q(2\xi) = \exp\,[i\varphi_n(\sigma_n + \xi)] \sum_{q=0}^{\infty} a_{qn-1} D_q(2\xi) \exp\,(iq\theta) \tag{53}$$

and because of the orthogonality property of the parabolic cylinder function

$$a_{qn} = \sum_{p=0}^{\infty} a_{pn-1} c_{pqn} \exp\,(-ip\theta), \tag{54}$$

where

$$c_{pqn} = \frac{\displaystyle\int_{-\infty}^{\infty} \exp\,[i\varphi_n(\sigma_n + \xi)] D_p(2\xi) D_q(2\xi)\, d\xi}{\displaystyle\int_{-\infty}^{\infty} D_q^2(2\xi)\, d\xi} \tag{55}$$

is the coupling coefficient between the pth and qth mode at the nth lens.

We are interested only in small lens distortions and small beam departure from ideal; therefore,

$$\begin{gathered} |\,\varphi_n(\sigma_n + \xi)\,| \ll \pi \\ a_{qn} \ll 1 \quad \text{for} \quad q > 0. \end{gathered} \tag{56}$$

Accordingly, keeping only first-order perturbation terms and expanding $\varphi_n(\sigma_n+\xi)$ in Taylor's series, we obtain for (54) and (55)

$$a_{qn} = \sum_{\nu=1}^{n} c_{oq\nu} \exp\,[iq(\nu - n)\theta] \tag{57}$$

and

$$c_{oq\nu} = \begin{cases} 1 & \text{if} \quad q = 0, \\ \dfrac{i}{2^q q!} \displaystyle\sum_{\mu=0}^{\infty} \frac{1}{2^{3\mu}\mu!} \frac{\partial^{q+2\mu}\varphi_\nu(\sigma_\nu)}{\partial\sigma_\nu^{q+2\mu}} & \text{if} \quad q > 0. \end{cases} \tag{58}$$

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